International Journal of Scientific & Engineering Research Volume 11, Issue 4, April-2020
ISSN 2229-5518

Optical Tomography in Medical Imaging and Diagnostic Engineering

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Abstract— Diseases are a part of human life. Being a human, it is our entire responsibility to look after the patients. With this thinking, medical field is always evolving so that maximum useful contribution could be delivered to serve mankind. In the medical field, medical imaging plays an important role to study and to diagnose different diseases. Requirement of diagnosis plays a role of catalyst towards advancement in the techniques of medical imaging. Optical tomography is one of the invasive emerging technique that is not only low cost but is also effective for several attempts because of its non-harmful nature of radiations. In this research, comprehensive study of optical tomography is conducted. Brief overview of different imaging techniques are presented so that importance of optical tomography could be highlighted on the basis of comparison. Photonics is the main component of optical tomography. Therefore, mathematical formulation of forward and inverse problems of optical imaging is derived. In addition to this, different methods are compared for the solution of photon transport model of optical tomography. Simulation study is added to validate the importance of optical tomography in medical imaging and diagnostic engineering.

Index Terms— Optical Tomography (OT), Medical Imaging, Medical Diagnosis, Forward problem, Inverse Problem, Photon Transport Model, Image Reconstruction.

1 Introduction

Optical tomography is a non-invasive technique in which visible or near infrared radiation is applied to analyze biological tissues. This technique is becoming famous among scientists and engineers because of its low-cost as compared to other imaging techniques and has a benefit of revealing in-depth anatomical information. In this research, technique of optical tomography is analyzed in detail for medical imaging and diagnostic engineering [1], [2].

Medial imagining is rapidly growing field of interdisciplinary research nowadays. Scientists and engineers are devising new techniques for diagnosis and treatment of diseases. In medical imaging, image of inner body is extracted for analysis and for examining medical disorder as well as visual representation of some organs and tissues [1].

Earlier, the most reliable way of diagnosis was biopsy in which a sample of tissue was obtained from body for in-depth examination and visualization. Invasive surgery was required to obtain tissue sample. This process was painful as well as risky. Therefore, there was an increasing need for new diagnostic techniques that would be capable of extracting tissue information without any surgical operation. X-ray was the first imaging technique. Later on, several medical techniques are developed such as magnetic resonance imaging (MRI), computed tomography (CT), positron emission tomography (PET) and Nuclear Medicine etc. These are very beneficial techniques but have also harmful effects on human body due to absorption of harmful extra high radiations and contrast agents on body and therefore, not advised to be used on regular basis. On the other hand, in some cases, like breast cancer diagnosis or brain tumors diagnosis, CT or ultrasound imaging are very costly and they need expert radiologist otherwise relatively poor precision results in disaster. Therefore, scientists and researchers are searching for an imaging technique that can non-invasively differentiate between malignant and healthy tissues [1], [2].

It is found from research that optical tomography is the best alternative of all these techniques. Practically, in optical tomography, light is guided by fiber optics (which are placed very close to the subject) to the surface of the object under test and detecting fibers (which are placed very close to the object) are used to measure the trans-illuminated or backscattered light. This trans-illuminated light is scattered through the multiple tissues layers and structures present inside the tissues. The trans-illuminated light is then converted into a series of voltages for amplification, filtration and digitization employed for image reconstruction. Information is dependent on either time varying intensities or steady state complex intensities of light that has measurable amplitude and phase. The image reconstruction with optical tomography is a great challenge for the scientists and researchers. Crosstalk noises often affect the useful information. Lot of research work is required to implement this technique in its true senses. In this research, optimization and reconstruction of image are discussed to make this technique more useful [1], [2].

This research paper consists of following sections: Section 1 contains the introduction. Review of different medical imaging methods is presented in Section 2. Section 3 covers the photon theory. Photon transport model of optical tomography is explained in Section 4. Numerical solution of diffusion equation for optical tomography is discussed in Section 5. Forward and inverse problems for image reconstruction in optical tomography is explained in Section 6. Section 7 covers the simulation for the validation of applications of optical tomography. Conclusion of research is presented in Section 8.

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2 Review of Different Medical Imaging Methods

Following are the different medical imaging techniques found in literature and in real practices.

2.1 X-Rays Tomography

X-rays radiography is a technique in which x ray beam is projected on an object and detected either on photographic film or in an ionization chamber. A numerous amount of x rays is absorbed by a subject under observation, which is dependent on the density and composition of the subject. The detector behind the subject measured that absorption of x-rays and construct an image or structure of the object [3], [4], [5].

The contrast in X-ray image occur between the tissues due to differential attenuation of X-ray in the body. They are highly attenuated by bones but readily penetrate soft tissue. The information is extracted from the transmitted photon that pass straight through the body. The scatter photons are also detected by the detection apparatus but they cause the damaging effect to the body structure such as low contrast and image relics. To improve image contrast some radio opaque contrasting materials such as boron and barium compound are used particularly in soft tissue image construction [5].

Skeleton imaging in sometime necessary to analyze the fracture and re-setting broken bones.it can also be used to observe blood flow in the brain and heart as well as the peripheral venous and arterial system. X-ray can also detect small lesion in breast tissue.it can image the liver bladder, abdomen and pelvis and can detect disease.it can also scans and detect abnormality within intestine and kidney [5].

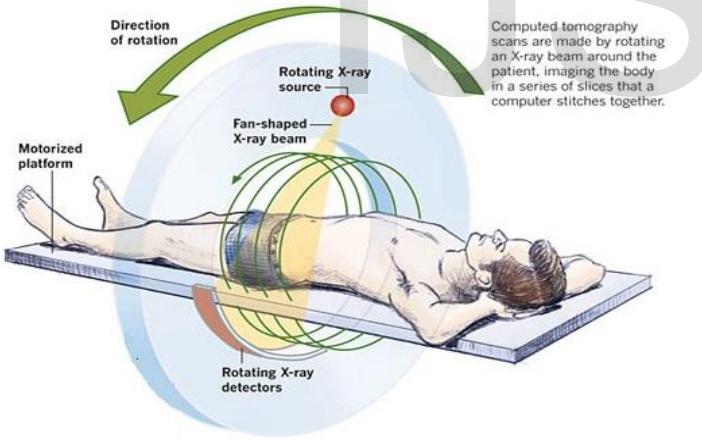

Fig. 1: Basic diagram of Computed X-ray Tomography.

2.2 Computed Tomography (CT)

Computed tomography or CT Scans is the modified form of X-ray. It is also called CAT scanning (computerized axial tomography). The word tomography is originated from the word "tomos" means slice/section and "graphe" means drawing in Greek Language. So, it provides a different form of imaging called cross sectional imaging. In this technique a planar slice of the body is subjected and examine from a multiple angle using cone type x ray beam. The x ray beam and measurement detectors are rotated around the patient at same angular speed and provide multiple snapshot of the object with wide range of different angles as shown in Fig. 1.

CT Scans are used to analyze skull fracture, brain damage, stroke and hemorrhage. They are used to diagnose both malignances and diffused disease of lungs such as silicosis, cystic fibrosis and emphysema and to diagnose abdominal tumors and ulceration of liver. Blood circulations of body can also be accomplished by CT Scans with high resolution image [6].

As X-ray and CT Scans both use ionizing radiations which cause Tissues damage. Therefore, exposure to X-ray should kept minimum to human body.

2.3 Ultra Sound Imaging

Ultrasonic imaging is a non-invasive and moderately inexpensive diagnostic technique to capture real time functional imaging. It primarily uses high frequency (around 10MHz) sound waves to insight body. Image are constructed from sound waves that are backscattered at boundaries of tissues and from a small structure within tissues [4], [5], [6], [7], [8], [9].

During ultrasonic image a short narrow beam of ultrasound/acoustic waves which corresponds to the upper limit of sounds audible to humans (>20000 Hz) is emitted from transducer which is placed closed contact with human skin. This beam propagates through tissue and is scattered by tissues and organs structures boundaries. The fraction of ultrasound that is backscattered along its original path is received by transducer as an echo. The transducer converts the backscattered pulsate in to a series of voltages, which are improved, filtered and digitized promptly to produce a moving image.

The depth and location of tissue interfaces are gathered from the time passed between the emitted pulse and the received echo, and the transmission velocity of sound in the transmitting medium. Low frequency (1-3MHz) ultrasound penetrates deep into the human body and is consequently used to image deep lying structures such as internal organs. High frequency (5-10MHz) ultrasound has high basic spatial resolution but low penetration depth and is therefore used to image tissue near the surface of the body.

Ultrasonic imaging uses non-ionizing radiation and is therefore utilized in sensitive regions such as to visualize the eye and the pregnant abdomen. There are some serious concerns regarding the radiation effect of ultrasound waves on developing tissues so acoustic energy and contact time are always kept to a minimum. Clinical applications using ultrasonic radiation are limited because of poor tissue contrast and the ability of gas and bone to hamper ultrasound [8], [9].

Ultrasound is used to check foetal (baby development) health. Constraints such as head size, the condition of the brain ventricles, blood velocity, and spinal condition are generally monitored to check whether a foetus is healthy or not. It is also used in combination with x-ray mammography to diagnose breast cancer [5], [8], [10].

Ultrasonic techniques are also used to diagnose diseases such as spewing, inborn heart disease and cardiac tumors. Further applications of ultra sound imagining are intra-abdominal imaging of kidneys, gallbladder liver and spleen [5], [8], [9].

2.4 Magnetic Resonance Imaging (MRI)

It is an imaging method generally used in medical imaging to construct high quality images of the organs inside of the human body. It is based on the principle of nuclear magnetic resonance-NMR, a spectroscopic technique which is used to analyze

the microscopic chemical and physical gen about molecules.

MRI produces an image of the nuclear magnetic resonance signals in a thin slice through the human body. The image formed is nearly equivalent to cutting off the anatomy above the slice and below the slice. The slice is consisting of a number of volume elements or voxels having volume of about 2mm3. MRI image is made of a several pixels. The intensity of a pixel is proportional to nuclear magnetic resonance signal intensity of the corresponding volume element or voxel of the object being imaged [11]. The pictorial illustration of the MRI imaging is as shown in Fig 2.

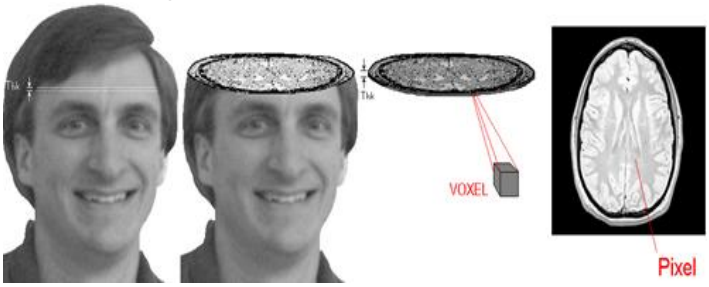

Fig. 2. MRI technique, representation of slice, voxel and pixel.

MRI primarily images the nuclear magnetic resonance (NMR) signal from the hydrogen nuclei/proton [11]. In the absence of magnetic field, protons are arbitrarily orientated. When a powerful, external magnetic field is applied on the human body it causes the protons to align in to parallel or anti-parallel shapes. In the lower energy state, the nuclei align in parallel to the direction of the applied magnetic field, while in the higher energy state, the nuclei align anti parallel to the field. The angular momentum of the arranged protons roots them to precession around the static magnetic field with a frequency that depends on magnetic field strength. To ensure that precession is coherent a weak radio-frequency field is applied on body. Which causing the photons to detect as an induced voltage in a tuned detector coil. This analogue signal is then digitized and is converted into a spatial image by using inverse Fourier transform [9], [12], [13]. MRI can detect changes in relaxation times, and concentrations of water and mineral, all of which can be signs of disease.

The uses of MRI are Brain Imaging, Liver and the Reticuloendothelial System imaging, Musculoskeletal System imaging, Cardiac System imaging [5],[9]

The large and expensive equipment used, and the need to keep a patient motionless due to the slow chronological resolution of MRI acquisition also limit its appropriateness [9].

2.5. Nuclear Medicine

Nuclear medicine is also a non-invasive technique in which measurements are performed with external source of radiations in such a manner that it does not allow the radio nuclide measurement to isolate from surrounding body tissues or cross-talk from radio nuclide in non-target regions [14]. It uses small amount of radioactive material which is injected into the bloodstreams to construct the image of particular area of the body via special cameras to detect the gamma rays energy emitted from radiotracer traveling in bloodstreams.

The first true gamma camera was developed by the scientist Hal Anger in late 1950, which was the base to make a 2D planar detector to produce two-dimensional projection image without scanning. When it is designed for tomography, it is called a Single Photon Emission Computed Tomography (SPECT) camera [15].

The radiopharmaceutical turns the system or organ being studied into a radioactive source. When radioactive compound decays the gamma radiation is detected externally using gamma camera. To realize the position of the source, a collimator is placed between the body and the detector to record only gamma radiation with angles close to 90o to the detector plane. A scintillation crystal at that point converts the gamma-ray energy that passes through the collimator into light, which in order is converted into an electrical impulse using photomultiplier tubes. The electrical impulse made from the spatial distribution or the radiopharmaceutical, is analyzed by computational tackle to form an image. Abnormal tissue scattering or an increase or decrease in the rate at which the radiopharmaceuticals store in particular tissues, organs, or systems are symptomatic of disease [9], [14], [15].

There are two nuclear imaging methods, both of which depend on the type of radioactive isotope used.

a. SPECT- (Single Photon Emission Computed Tomography)
b. PET- (Positron Emission Tomography)

2.5.1. Single Photon Emission Computed Tomography

SPECT uses single photon emitter as a radioactive source and therefore SPECT scans are performed from various angles using one or more gamma cameras mounted on a computer controlled rotating scaffold. The scaffold circles the patient at predetermined increments to form a two-dimensional image of the tracer distribution [5], [16], [17], [18].

2.5.1. Positron Emission Tomography

PET Scans uses positron emitting radiopharmaceuticals. It detects pairs of gamma rays that emitted indirectly by a positron-emitting radioactive tracer. The gamma radiations produce due to electron- positron annihilation. The acceptance of annihilation event is conditionally with the detection of both photons in coincidence by an array of detectors around the body. Therefore, collimators are not used in PET. The detection of coincident photons is used to locate the line, along which the original decay occurred, which is consequently mapped as a two dimensional image [5], [9], [15], [16], [17]. The major application of PET Scans in medical imaging are Brain Imaging, Cardiac imaging and Tumor and cancer imaging [5], [9], [17], [18].

2.6. Optical Tomography

Optical tomography (OT) is also a non-invasive technique in which we use visible or near infrared radiation to analyze biological media. Apart from different medical imaging technology, it has captured the attention of scientists and engineers due to its low-cost alternative to other imaging techniques, with an advantage of providing anatomical information [9], [19]. Practically in optical tomography light is guided by fiber optics (which are placed very close to the object) to the surface of the

object and detecting fibers (which are also very close to the object) are used to measure the Trans illuminated or backscattered light. This trans-illuminated light is scattered through the multiple tissues layers and structures present inside the tissues. The Trans illuminated light then converted into a series of voltages to amplify, filtered and digitized to construct an image. The data is developed either through time varying intensities or steady state complex intensities of light that has measurable amplitude and phase [5], [9], [19], [20], [21]. The basic sematic diagram of optical tomography is as show in Fig. 3.

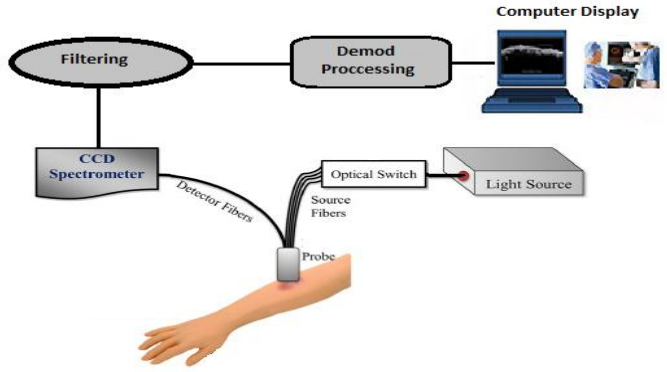

Fig. 3. The Schematic of an optical tomographic system

The optical properties of the tissue are mainly confined in scattering and absorption coefficients of light and are exploited to provide qualitative and quantitative images. The differences in optical properties between healthy and irrational or damage tissue provide different contrast when imaging disease. The healthy tissues properties are already measured and stored to compare with unhealthy tissue [11].

Medical application of optical tomography are brain imaging, skin and breast cancer imaging. Osteoarthritis, rheumatoid arthritis and diabetes treatment are also done with optical tomography [9]

Optical tomography has a great advantage over others non-invasive techniques such as X-ray, MRI, Nuclear Medicine regarding cost and noxious effect. The large dosage of light exposure to the body has no harmful effect on tissues rather than ionizing or magnetic affects. The absorption coefficient of near infrared radiation is very low that's why it can penetrate intensely into the tissues and can provide deep tomographic imaging experiences. While the scattering coefficient is high which causes deprivation in image resolution and contrast. The diverse/heterogeneous properties of tissue also cause scattering coefficient high [22].

Optical tomography has limited development considering lack of suitable instrumentations and classy scattering models. The image reconstruction become a non-linear problem as a result of dominating scatter and direct reconstruction methods such as back projection can't be used. As an alternative non-linear, iterative inversion reconstruction schemes are often working. But the computational cost of running inversion calculations can take hours, limiting the technique to simplistic models and conditions that generalize a more complicated medical system. It is thus required to use optical tomography in accordance with a second imaging technique, like MRI or CT, to monitor changes in critical parameters or to assist ongoing diagnosis and research. A method like MRI can provide an initial high-resolution map of tissue distribution, which can be used to guide the computational reconstruction of the low cost, non-invasive optical image [19], [21], [23], [24], [25], [26], [27].

3 Photon Theory

3.1. Properties of Light

The optical properties of tissues are generally describing in term of absorption, scatter, anisotropy and refractive index. All of these parameters are dependent on the wavelength of the light penetrating in the tissues. Other optical properties of tissue are fluorescence and inelastic scatter.

3.1.1. Absorption of light

When light radiation is incident on substance confined of discrete electrical charges, the charges are forced to oscillate at the frequency of the incident electric field. The frequencies ranges of radiation in the infrared region of the electromagnetic spectrum is similar to the natural frequencies at which atoms or molecules will vibrate in the presence of an applied field. Thus, when infrared radiation is incident on medium, resonance will occur about the natural frequencies, by which energy is transferred from the incident field to the medium and its amplitude of vibration is significantly increased. Though the lifetime of the excited state is around seven to ten seconds, the atoms or molecules will usually lose their energy by colliding with one another within ten to twelve seconds, so raising the kinetic energy of the other particles involved in the collisions. Hence, the energy related with the incident field is most often dissipated as heat within the medium. This process is known as absorption [9]. The intensity of the light beam crossing the medium is decreased. This relationship was first found by Bouguer (1729). After few years, Lambert (1760) also derived the mathematical expression for this relationship, which is known as the Lambert-Bouguer law as given below:

$$\frac{dI}{I} = \mu_a dl \qquad (1)$$

This expression describes that each consecutive layer (defines as dl) of the medium absorbs the same amount (called $\frac{dI}{I}$) of the intensity I over a constant known as absorption coefficient μ_a and is expressed as the product of the atomic number or particle density contained in the material, $\mathcal{A}$(r), and the microscopic absorption cross-section $\sigma_a(E)$ and is shown in Fig. 4:

$$\mu_a(\boldsymbol{r}, E) = \mathcal{A}(\boldsymbol{r}) * \sigma_a(E) \qquad (2)$$

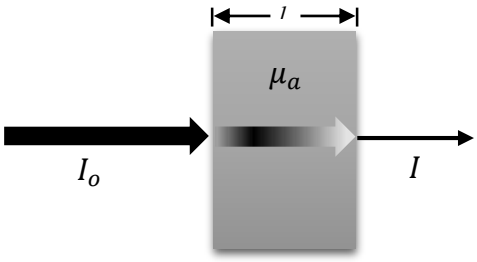

Fig. 4. Absorption of light through a non-scattering medium

Where $\boldsymbol{r}$ represents a position variable and E represents the

energy variable. The reciprocal of $^1/_{\mu_a}(\boldsymbol{r}, E)$ is known as the absorption path length and also called the mean free path, in which a photon travel between subsequent absorption events [9]. This can be illustrated in Fig. 4. For an incident intensity I_o, the intensity I which is transmitted through a distance l will be decreased and expressed as

$$I = I_o e^{-\mu_a l} \quad (3)$$

The absorption coefficient μa of a medium is due to a number of absorbing chromophores mixed together. The individual extinguishing/extinction coefficients of each chromophore denote its absorption at a particular concentration. The absorption coefficient can be stated as the sum of the products of the concentration, of each chromophore Cn with its extinction coefficient Ɛn and be written as

$$\mu_a = \sum_n c_n * \varepsilon_n \quad (4)$$

Therefore, equation (3) becomes:

$$I = I_o e^{-(\sum_n c_n * \varepsilon_n) l} \quad (5)$$

This expression is called Beer-Lambert Equation and is only valid under the conditions that "the light entering into the medium must be monochromatic and perfectly isotropic, and the medium is also purely and uniformly absorbing". Therefore, there must be some errors when applying the law to practical measurements because monochromatic property of light is practically non-achievable as even lasers are not perfectly monochromatic [9], [22].

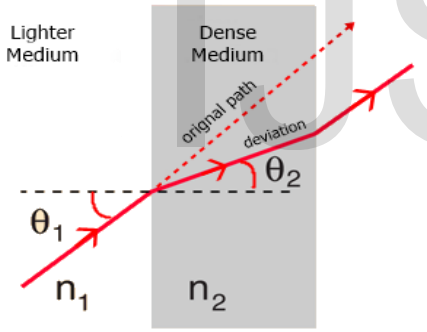

Fig. 5. Refraction at the interface between two mediums with different refractive indices.

3.1.2. Refractive Index

In optics, the refractive index of the material defines how light propagates through that medium. The speed change when light passes between media of different intensities causes its direction to change as shown in Fig. 5 [28], [29].

The refractive index is defined as the ratio between the speed of light in vacuum to the speed of light in medium and is denoted by "n":

$$n = \frac{c}{v} \quad (6)$$

Here c means the speed of light in vacuum and v represent the speed of light in other medium.

Snell's Law also stated that:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad (7)$$

Where the sign n denote the refractive index of the medium and θ denote the angle between the propagation direction and normal to the boundary of two mediums. The refractive index of the medium is depending on the number of particles/molecules per unit volume and polarizability of the medium.

The refractive index of the medium changes with the wavelength and frequency of the incident light. Because of this property of medium, the white light when fall on the medium divide into its constituent spectral colors, which is called dispersion of white light. Theory has shown, that when light passes through a medium, some part of it will always be attenuated. Thus, the complex refractive index has taken into consideration.

$$N = n + ik \quad (8)$$

Here n is the real part of the equation and is define the speed of the incident light and the imaginary part 'k' tell the absorption of the incident light as it propagates through the medium [28], [29].

3.1.3. Scattering of Light

When an incident light propagates through medium, the charged particles present in a medium are set into oscillatory motion due to the electric field of the incident wave and some of the light particles re-emit from the medium with same frequency as the incident wave particles, this is called scattering of light.

In an optically dense or homogeneous medium, atoms or molecules of the medium may scattered the incident radiations in all directions. The angle between the scattered light relative to the incident light wave will depend on the frequency of the incident light and the total scattered field which happen in all direction will be the sum of all the scattered wavelets propagating in its particular different direction. The scattered waves will affect the incident wave resulting in modifying its phase and hence the velocity of the light through the medium also change.

There are many theories that explain the effect of scattering by different shapes and sizes of the medium particles. According to nature of scattering, these theories are divided into two categories, one is called single scattering theory and other is called multiple scattering theory [9], [12]. The exponential relationship which is formed by the scientist can be defined as the scattered light intensity I as shown in Fig. 6, comparative to the incident beam of light with intensity Io, when diffused at a length l through an absorbing medium in which only single scattering occurs will be equal to the mathematical expression given below:

$$I = I_o e^{-\mu_t l} \quad (9)$$

Where μ_t is called total attenuation of light given by:

$$\mu_t = \mu_a + \mu_s \quad (10)$$

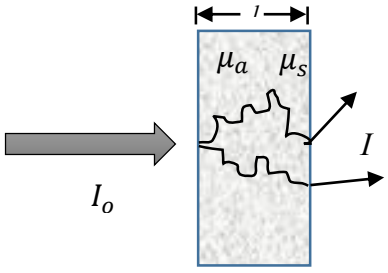

Fig. 6. Scattering of light in scattering medium.

The term μ_s is called scattering coefficient and is expressed in terms of atomic number or particle A(r), and microscopic

scattering cross section:

$$\mu_s(\boldsymbol{r}, E) = \mathcal{A}(\boldsymbol{r}) * \sigma_s(E) \quad (11)$$

The reciprocal, $1/\mu s(r, E)$, is named as the scattering path length and equal to the average distance, a photon travels between two consecutive scattering events. Practically this assumption is invalid because multiple scattering occurs in biological tissues and the effect of multiple scattering cannot be ignored [22].

3.1.4. Anisotropy

The scattering of light as discussed earlier is different from absorption, because in scattering the photons are not travel in a straight line in the medium and can be scattered anywhere in any angle in three dimensions If a photon is incident along a unit vector say a then the probability that it is scattered into different direction say b can be defined by a phase function $f(a, b)$ as shown in Fig. 7.

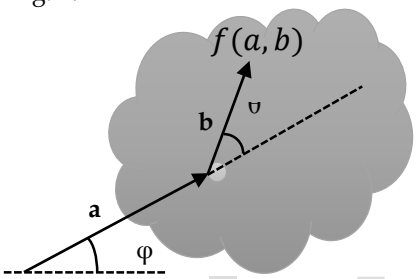

Fig. 7. Phase function f (a, b) representation

It is assumed that the phase function is in the form of the angle between the incident and scattered light in random scattering media which is independent of the orientation of the scatterer. But this assumption does not hold for some tissues in biological media such as muscle where scattering properties change in different orientations. The anisotropy is then defined as the mean cosine of the scattering angle and expressed by the expression:

$$g = \int_{4\pi} d(\cos(\theta)\, cos\theta\, d\theta \quad (12)$$

Where g is called anisotropy factor which is depends on the size of the scatterer, shape of the scatterer and refractive index mismatches of the scatterer, if g is equal to zero then the scatter is isotropic but if g is equal to one, the light will be entirely forward scattered and if g is equal to -1, the light is entirely backscattered [30].

4 Photon Transport Model

Photon propagates through the medium in a deterministic and foreseeable, described by the Boltzmann transport equation or radiative transfer equation (RTE). It is used in a more general sense and refer to any kinetic energy equation that describe the change of energy radiance in time as a result of change in energy flow. Basically, it is derived by considering the effect of gains and losses of the intensity of photon travelling in the mediums such as tissues.

In a nutshell, the radiative transfer equation states that a beam of light losses energy through absorption and scattering away from the beam and gains energy from the light source in the medium and scattering absorbed toward the beam [19], [24], [25], [31], [32] [33] [34] [35]. The radiative transfer equation (RTE) for photons which are travelling from the point r in the direction ŝ with a specific intensity I(r, ŝ, t) at a time t will be

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \hat{\mathrm{s}}.\boldsymbol{\nabla} + \mu_t(r)\right) I(r, \hat{\mathrm{s}}, t) = \mu_s(r) * \int_{4\pi} p(\hat{\mathrm{s}}.\hat{\mathrm{s}}') I(r, \hat{\mathrm{s}}', t) d\hat{\mathrm{s}}' + Q(r, \hat{\mathrm{s}}, t) \quad (13)$$

The explanation of each term of the radiative equation is given in Table 1.

TABLE 1
TERMS OF RADIATIVE TRANSFER EQUATION

Sr. No.	Terms	Explanation
1	$\frac{1}{c}\frac{\partial I(r, \hat{\mathrm{s}}, t)}{\partial t}$	No. of photons entering in the volume of interest minus No. of photons emitting per unit time.
2	$\hat{\mathrm{s}}.\nabla I(r, \hat{\mathrm{s}}, t)$	The photon flux at point *r* travelling in the given direction in given time.
3	$\mu_t(r) I(r, \hat{\mathrm{s}}, t)$	The reduction of Intensity of light (attenuation) in the volume of interest.
4	$\mu_t(r) = \mu_a(r) + \mu_s(r)$	Total attenuation of light that is the sum of the absorption coefficients and the scattering coefficients.
5	$p(\hat{\mathrm{s}}.\hat{\mathrm{s}}')$	It is the normalized phase function which represent the probability density of scattering of light from direction ŝ to direction ŝ'
6	$\int_{4\pi}^{0} p(\hat{\mathrm{s}}.\hat{\mathrm{s}}') I(r, \hat{\mathrm{s}}', t) d\hat{\mathrm{s}}'$	The No. of photons scattered from direction ŝ into another direction ŝ′
7	$Q(r, \hat{\mathrm{s}}, t)$	The source term which describe the No. of photons emitted at point *r* per unit time in direction ŝ

The quantities which are derived from the equation are photon density and photon flux/current is:

$$\Phi(r, t) = \int_{4\pi} I(r, \hat{\mathrm{s}}, t)\, d\hat{\mathrm{s}} \quad (14)$$

$$J(r, t) = \int_{4\pi} \hat{\mathrm{s}}.I(r, \hat{\mathrm{s}}, t)\, d\hat{\mathrm{s}} \quad (15)$$

These are the measurable parameters for μa and μ's [19]. The

main approximation which we apply, are the P1 and PN approximation.

4.1. P1 and PN Approximation

To solve the RTE in less computation time, some approximations and simplifications are required The PN approximation is derived if the first N spherical harmonics is taken, which gives $(N+1)^2$ coupled partial differential equations (PDEs). By taking the first spherical harmonic, four PDEs are obtained, combined to give P1 approximation. Substitution of these terms into the equations (14) and (15) for the P1 approximation leads to the following expressions [22]:

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\mu_a(r)\right)\Phi(r,t)+\nabla.J(r,t)=Q_o(r,t) \tag{16}$$

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{1}{3\kappa(r)}\right)J(r,t)+\frac{1}{3}*\nabla.\Phi(r,t)=Q_1(r,t) \tag{17}$$

Where Q0 is isotropic component of source, Q1 is anisotropic dipole-like component of source. κ = Diffusion coefficient = $\frac{1}{3(\mu_a+\mu_s')}$, and $\mu_s'=(1-p_1)\mu_s$. These expresseions are connecting the measurable quantity photon density Φ and photon current J to the optical properties μ_a and κ. The prominent thing is that the equation (16) contains isotropic source term while the equation (17) contains anisotropic source term.

4.2. The Diffusion Approximation

By making further assumption to neglect the effect of scattering it is assumed that the source is isotropic (i.e.; its properties are independent of medium geometry) and the time rate of change of photon flux are also too slow. i.e.; $q_1=0$ and $\frac{\partial J}{\partial t}=0$. Therefore, by putting these values in the equation (17):

$$J(r,t)=-\kappa(r)*\nabla\Phi(r,t) \tag{18}$$

This expression is also called Fick's Law which states that the rate of diffusion of gases and molecules is directly proportional to the product of diffusion coefficient of the medium and concentration gradient. By substitution the expression (18) in expression (16), the new expression will be like that:

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\mu_a(r)\right)\Phi(r,t)-\nabla.\kappa(r)*\nabla\Phi(r,t)=Q_o(r,t) \tag{19}$$

This expression is modified radiative transfer equation and is also an energy conservation equation or diffusion equation.

$$\underbrace{\frac{1}{c}\times\frac{\partial\Phi(r,t)}{\partial t}}_{\substack{Flow\\Energy}}+\underbrace{\mu_a(r)\times\Phi(r,t)}_{\substack{\text{Absorption}\\\text{Energy}}}+\underbrace{(-\nabla.\kappa(r)\times\nabla\Phi(r,t))}_{\substack{\text{Diffusion}\\\text{Energy}}}=\underbrace{q_o(r,t)}_{\substack{\text{Source}\\\text{Energy}}} \tag{20}$$

The diffusion approximation assumes that:

a. The source's angular dependence, specific intensity of the source and phase function of the scattering parameter are uniform in order to model as first order spherical harmonics. In practical aspect, it means that photons must be isotopically scattered.
b. The time rate of change of the photon flux is approaches to zero. This indicates, the distribution of light within the object is rapid, which is practically invalid for time dependent case.

To achieve these condition, this will be defined by assuming $\mu_s' \gg \mu_a$[19].

4.3. Boundary Conditions

To make diffusion equation specific to boundary value problem, boundary condition is applied on it. The first condition apply on it is that no photons travel in an inward direction at the boundary except source photons that is $\phi(r,\hat{s},\boldsymbol{t})$ is equal to zero if $\hat{s}.\hat{n}\leq\mathbf{0}$, where $\hat{\boldsymbol{n}}$ is the outer normal to surface at point r. The diffusion equation also can't satisfy this condition precisely, rather we take the assumption that the total inward directed current is zero. [22]

4.4. Limitations of Diffusion Approximation

In addition to the boundary condition $\mu_s' \gg \mu_a$, there are also two main problems when using the diffusion approximation, one is: clear regions have no scattering and diffusion parameters and second is: there are jumps in the medium. If medium have piecewise continuous regions, then the boundary conditions of the diffusion equation cannot be exactly meet at the interfaces [19]. In these situations, higher-order approximations is required. Another weighty problem is non-scattering regions. Many numerical solutions to the diffusion equation, except for Monte Carlo, cannot handle this case, so special methods need to be developed to address such type of problem [19].

5 Numerical Solutions of Diffusion Equation

There are several numerical methods that may be used to solve the diffusion equation. These approaches are a lot more versatile than trying to derive an analytical solution. Analytical solutions of the diffusion equation uses Greens functions. However, these functions involve much computational time and physical memory of the system and cannot be applicable for wide range of geometries.

5.1. Finite Element Method (FEM)

This method was formerly developed by Clough (1960) to study the stress analysis in complex air-frame structures and was more developed by Zienkiewicz and Cheung to the general field of continuum mechanics in 1965. Because of its diversity and flexibility as an analysis tool, it takes a lot of attention [36], [37].

The finite element method divides the solution region in many small, interconnected, sub-regions or elements which are called nodes and gives a piece-wise approximation to the governing equations those make the sub regions. These governing equations are the complex partial differential equations which are compacted to either linear or nonlinear simultaneous equations [37].

Mathematically it divides the domain say Ω, into a finite number of elements which have a specified number of corners or nodes. The photon density $\Phi(r,t)$ is thus calculated at each node and summed all over N nodes in the domain. The mathematical representation of the photon density $\Phi(r,t)$ using the piecewise function is given below:

$$\Phi^h(r,t) = \sum_{i}^{N} \Phi_i(t) u_i(r) \tag{21}$$

This expression signifies the splitting of the domain into a finite number of subdomains called elements. Each element has nodes at its corners, so that the domain has N nodes as shown in Fig. 8. [37], [38]. The solution can also be mapped to a local basis by using new approximation function:

$$u_i(r) = \sum_{p=1}^{n} u_p v_p \tag{22}$$

Here v_p is the new local basis that generally smooth the solution u_i which is important for iterative methods. A number of possible local basis functions are illustrated in Fig 9. [38]

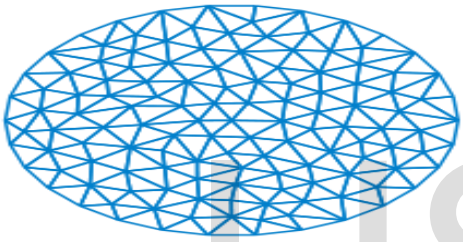

Fig. 8. A finite element mesh used for finite element. Each triangle is an element and each vertex is a node

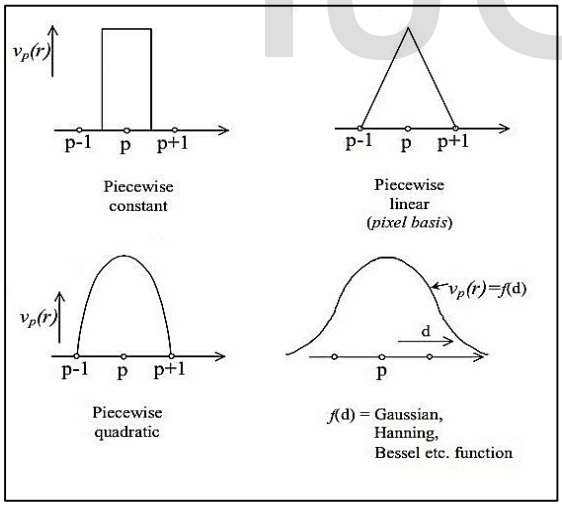

Fig. 9. Different type of local basis functions. .

5.2. Finite Difference Method

It is a special case of finite element method where a regular grid is used instead of a more versatile mesh. The finite difference is well-designed and theoretically simple method that gives a point-wise approximation to the governing equations which are also linear equations. The finite difference model, formed by writing the difference equations for an array of grid points, can be improved if number of points are increased [37], [38].

5.3. Finite Volume Method

Finite Volume Method is a more refined version of finite difference method and has become more popular in computational fluid dynamics study. It uses a volume integral formulation of the boundary problem with a finite partitioning set of volumes to discretize the governing equations It has different techniques to solve but the vertex-centered finite volume technique is popular and is very similar to the linear finite element method [37], [38].

5.4. Monte Carlo Modelling

The diffusion approximation is not valid where the condition μa << μ's does not hold. In these cases, an alternative model must be used if the propagation of light is to be properly modelled. Monte Carlo modelling (MC) model can be applied such type of model in efficiently manner.

In this method, a model is established with a precise geometry and distribution of optical parameters. When the model is formed, simulation is performed on it and photons are emitted one by one from a modelled source into the modeled system. As a photon propagate through that modeled system, the probability that it is scattered in any direction is combined with the probability that it will absorb in system. The direction in which the light is scattered is formerly determined from the modelled phase function. Each photon is assigned a 'weight value' that represents its geometrical probability of not having been absorbed. Thus, in the end the weights of all the photons which reach the detectors are summed up, and information about the distance that a photon has travelled through the medium can be retrieved.

For tomographic image reconstruction, where a photon can have several hundred interactions while propagates between a source and detector, and a large number of photons (about millions) are needed to obtain suitable statistics. As a result, other image reconstruction methods are preferred over Monte Carlos Modelling because of its high computational time.

6 Image Reconstruction

The objective of image reconstruction in optical tomography is to determine the optical properties (i.e. μ_a *and* μ_s) of a theoretical model which produces the same results that are obtained experimentally.

6.1. Forward Problem

The forward problem can be modelled by the mathematical expression given below:

$$y = Jx \tag{23}$$

Where y are the measurements which are constructed, x are the optical properties of the medium and J is the sensitivity matrix which is also known as the Jacobian of the forward operator or may also called the weight matrix. The measurements y is calculated by knowing the geometry and optical properties of the medium, the location of the sources and detectors which are placed around the medium and the transport of light according to the diffusion equation.

The objective of the forward problem is to extract model data for comparison with experimental data during image reconstruction, or it may use to generate simulated data to test reconstruction techniques or expected outcomes of a photon study. The solution of the forward problem definitely, depend on knowledge and determination of the sensitivity matrix J. The sensitivity matrix can be calculated numerically by using Finite Element Method as discussed earlier or analytically by Monte Carlo modelling, or experimentally by placing a small absorber (and/or scatterer) at discrete positions within the domain.

6.2. Inverse Problem

In inverse problem, it is recommended to find the unknown functions occurring in the formulation of the forward problem from the data. This can be mathematically expressed by the equation:

$$x = J^{-1}y \tag{24}$$

In this equation y represents the data types that are extracted in forward model, x represents the optical properties of the medium which are unknown now. To solve this equation, inverse of Jacobian matrix is required. There are two characteristics of the sensitivity matrix, which make difficulties in inversion process:

a. An ill-posed problem has no solutions in desired class or has many solution, or the solution procedure is unstable [39]. In this research, this means that there are a smaller number of independent measurements than unknown pixel values.
b. A system or matrix is called ill conditioned if some small perturbation/changes in the system causes a relatively large or very large change in the exact solution [40]. This condition is typically resulting in the amplification of both the measurement errors and numerical errors in inversion process.

According to ill-posed and ill-conditioned system type, image reconstruction is also dived into two major types.

6.3. Linear Image Reconstruction

This simple method incorporates the approach of x-ray Computed Tomography (CT scan) image reconstruction with the weight functions which is reason for the non-linear propagation of light compared to x-rays.

The inversion of the sensitivity matrix may be non-trivial, therefore, single value decomposition (SVD) and Tikonov regularization may be applied, but the latter is more commonly used in optical imaging. The Tikonov regularization is normally calculate the optical properties by the equation given below:

$$\Delta x = \left[J^T J + \tilde{\lambda} I\right]^{-1} J^T \Delta y \tag{25}$$

Where $\tilde{\lambda} = \lambda$*Fmax and Fmax is the maximum diagonal element value of the matrix J^T. J and λ is a regularization criterion which calculate the accuracy of the match between the model and the reconstructed data [41].

6.4. Nonlinear Image Reconstruction

Nonlinear image reconstruction is at the core of the medical imaging problems. The objective of nonlinear image reconstruction is to calculate the optical properties of the medium at each point within the model using measurements of light propagation from the tissue surface. The two discrete approaches that can be used for image reconstruction for nonlinear system or matrix are gradient-based reconstruction techniques and Newton-like methods [22], [41]. The image reconstructions can be constrained by using a-priori data and information [22], [41].

7. Simulation Analysis

Matlab is used to validate the performance of optical tomography in frequency domain. Different cases are made by varying the number of sources, detectors, properties of optical source. Table 2 presents the description of cases performed in this research.

TABLE 2
DESCRIPTION OF DIFFERENT CASES

Sr. No.	Parameters	Case 1	Case II	Case III
1	No. of sources	16	24	32
2	No. of detectors	16	24	32
3	Refractive index of medium	1.3	1.4	1.4
4	Absorption coefficient of medium	0.06 mm^{-1}	0.02 mm^{-1}	0.02 mm^{-1}
5	Scattering coefficient of medium	1	1-	1
6	Radius of the 1st inclusion:	2 mm	2 mm	1 mm
7	Absorption coefficients of inclusion	0.1mm^{-1}	0.1mm^{-1}	0.1mm^{-1}
8	Radius of the 2nd inclusion:	-	1 mm	1 mm
9	Absorption coefficients of inclusion	-	0.3mm^{-1}	0.06mm^{-1}
10	Radius of the 3rd inclusion	-	-	1mm
11	Absorption coefficients of inclusion	-	-	0.3mm^{-1}
12	Stopping criterion	1x 10^{-6}	1x 10^{-10}	1x 10^{-10}
13	Number of iteration	100	100	100

Simulations are performed up to 100 iterations. 100 iterations are performed with less computational time. Table 3 presents the results obtained through simulations. These experiments have very appreciating results demanding a very deep study in this field. It also showed that image reconstruction of optical tomography is similar to the other medical imaging techniques but has less simulation time. Moreover, the image

reconstruction of optical media has more anatomical data instead of other imaging techniques due to different absorptions and scattering properties of medium/layers of tissues It provides us full detail of layers due to change of absorption and scattering coefficient of different layers.

TABLE 3

IMAGE QUALITY TEST UNDER DIFFERENT CASES

Sr. No.	Test Performed	Case I	Case II	Case III
1	Average Difference	0.0094	0.0084	-0.0010
2	Mean Square Error	7.7031e-004	0.0015	0.0088
3	Peak Signal to Noise Ratio	79.2642	76.2682	78.6625
4	Mean Normalized Cross-Correlation	0.2165	0.4368	0.7120
5	Structural Content	18.883	3.9870	1.4345
6	Maximum Difference	0.0890	0.2271	0.1656
7	Normalized Absolute Error	0.8484	0.7545	0.9659

8. CONCLUSION

In this research, detailed analysis has been carried out for the application of optical tomography in medical imaging and diagnostic engineering. It is found from mathematical formulation that it is capable of diagnosing the abnormality in tissues with less complexity and in less computation time. No compromise policy over quality of images is found from simulation results irrespective of its number of sources and detectors and irrespective of optical properties of light. Most of the anatomical data is revealed with this medical imaging technique. Therefore, this technique is considered to be a major breakthrough in the field of medical diagnosis. In future, this technique can be tested on large scale imaging and prototype can be developed so that noninvasive approach of studying of images can be carried out without any fear of consequences on human body.

ACKNOWLEDGMENT

The authors are very thankful for the Department of Electrical Engineering, Air University, Islamabad, Pakistan and Department of Electrical Engineering, University of Lahore, Lahore, Pakistan for providing us an opportunity to conduct successfully research on applications of optical tomography in medical imaging and diagnostic engineering.

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